Stochastic growth of ion cyclotron and mirror waves in Earth's magnetosheath

Iver H. Cairns and K. A. Grubits
School of Physics, University of Sydney, NSW 2006, Australia
(Received 2 November 2000; published 24 October 2001)

Electromagnetic ion cyclotron and mirror waves in Earth's magnetosheath are bursty, have widely variable fields, and are unexpectedly persistent, properties difficult to reconcile with uniform secular growth. Here it is shown for specific periods that stochastic growth theory (SGT) quantitatively accounts for the functional form of the wave statistics and qualitatively explains the wave properties. The wave statistics are inconsistent with uniform secular growth or self-organized criticality, but nonlinear processes sometimes play a role at high fields. The results show SGT's relevance near marginal stability and suggest that it is widely relevant to space and astrophysical plasmas.

DOI: 10.1103/PhysRevE.64.056408 PACS number(s): 52.35.Hr, 52.35.Qz, 94.30.Tz, 94.30.Va

Earth's magnetosheath contains solar wind particles that have been slowed, compressed and heated at the bow shock, and are being deflected around Earth's magnetopause (Fig. 1) [1]. Since the magnetosheath is not in thermodynamic equilibrium, it contains high levels of waves from frequencies well below the proton gyrofrequency (a few Hz) to the electron plasma frequency f_{pe} ($\sim$20–50 kHz) [2–5]. Attention is focused here on mirror mode and electromagnetic ion cyclotron (EMIC) waves that have frequencies of order the proton gyrofrequency and below [2,4–7]. Observations show these waves to be dynamically important, quantitatively limiting proton temperature and pressure anisotropies [5–7]. The waves are usually very bursty, with widely varying magnetic fields that range from relative weakness to fields comparable to the background field $\mathbf{B}_b$ (the latter suggesting a possible role for nonlinear processes), and persist throughout much of the magnetosheath. Similar waves are observed in the magnetosheaths of Mercury, Venus, Mars, and the outer planets, as well as behind traveling interplanetary shocks. They are often observed in laboratory plasmas and are expected in astrophysical shock systems. In this paper it is demonstrated that a recent theory, stochastic growth theory (SGT) [8–13], can explain many properties of these waves, including their statistics, burstiness, widely variable fields, and persistence, with only a minor role for nonlinear processes.

The usual ''uniform secular'' model for wave growth in plasma physics [1,14] involves an initially homogeneous plasma in which waves grow exponentially in time with constant growth rate until saturated by nonlinear processes, while the wave growth relaxes the particle distribution towards marginal stability by reducing gradients in the velocity distribution function (quasilinear relaxation). At marginal stability, emission and absorption are balanced. Recently it has been demonstrated directly using particle measurements and linear instability theory that the EMIC and mirror waves in Earth's magnetosheath are close to marginal stability for proton temperature anisotropy instabilities [5–7]. These temperature anisotropies, between the temperatures perpendicular ($T_{p\perp}$) and parallel ($T_{p\parallel}$) to $\mathbf{B}_b$, originate partly at the bow shock, as a result of incomplete thermalization there, and partly in the magnetosheath, due to draping of the solar wind magnetic field over the magnetopause and escape of low pitch angle protons from the draping region (which reduces $T_{p\parallel}/T_{p\perp}$). This demonstration of closeness to marginal stability is very rare in space physics. However, the uniform secular growth model cannot explain many properties of the EMIC and mirror waves in planetary magnetosheaths, including their burstiness, highly variable magnetic field strengths, and their persistence far from the bow shock and throughout much of the magnetosheath. Similarly, the processes limiting the wave growth remain unknown. Until recently resolved by SGT, similar problems were posed by Langmuir-like waves and driving electron beams associated with type III solar radio bursts [8,9] and Earth's foreshock [11–13], while persistence of electron streams is a long standing astrophysical problem, i.e., in radio jets and pulsar magnetospheres.

SGT treats situations in which a source of free energy interacts with driven waves and the inhomogeneous ambient medium and evolves to a state in which (1) the particle distribution fluctuates stochastically about a state very close to time- and volume-averaged marginal stability and (2) the wave gain G is a stochastic variable in position and time. For waves with magnetic field B, G and the energy growth rate Γ are related to each other and a reference field B_0 by $B^2(t) = B_0^2 e^{G(t)} = B_0^2 \exp[\int dt\,\Gamma]$. SGT describes the random walk in G using the standard wave equations

$$\frac{dB(\mathbf{r},t)}{dt} = \frac{\Gamma(\mathbf{r},t)B(\mathbf{r},t)}{2}; \quad \frac{dG(\mathbf{r},t)}{dt} = \Gamma(\mathbf{r},t), \tag{1}$$

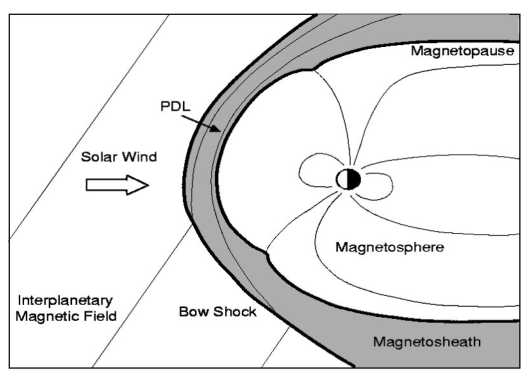

FIG. 1. Earth's magnetosheath lies between the magnetopause and the bow shock [1].

where $G(\mathbf{r},t)$ is a stochastic variable rather than the spatially homogeneous values $G(t)=\Gamma t$ and constant Γ assumed in the linear phase of the uniform secular model. SGT is then a natural theory for bursty waves with widely variable fields in time and space (due to the random walk in G and $\log B$) that exist together with the driving distribution unexpectedly far from its source (due to the closeness to marginal stability). Qualitatively, it is envisaged that preexisting inhomogeneities in the ambient plasma define favored sites for wave growth, in which wave-particle interactions inject fluctuations into the particle distribution that then evolve toward an SGT state.

This paper's primary aim is to establish that SGT quantitatively accounts for the functional form of the wave statistics, and so can qualitatively explain the growth and properties of mirror and EMIC waves in Earth's magnetosheath. This is important for several reasons:

(1) It is the first detailed explanation for these waves' burstiness, distribution of field strengths, and persistence. In contrast, the uniform secular model and self-organized criticality (SOC) [15] are demonstrably inconsistent with the observed wave statistics.

(2) These waves provide excellent opportunities to test SGT's relevance in a situation independently shown to be close to marginal stability [5,7], and so where it is favorable (but not certain), that SGT will apply.

(3) Simultaneously, this is the first application of SGT to primarily magnetic waves, waves driven by ions, and to a temperature anisotropy instability.

The second aim is to argue strongly, based on these results and SGT's previous successes for very different electron-driven waves [9,11–13], that SGT should be considered widely applicable in space plasmas and, by extension, in astrophysics. We also present weak evidence that nonlinear processes coexist at high fields with SGT for both EMIC and mirror waves.

Via the central limit theorem, the most fundamental and testable prediction of SGT for relatively simple systems is that the probability distributions $P(G)\propto P(\log B)$ should be Gaussian in G (lognormal in B) [8–12]:

$$P(\log B)=(\sigma\sqrt{2\pi})^{-1}e^{-(\log B-\mu)^2/2\sigma^2}, \tag{2}$$

where μ and σ are the average and standard deviation of $\log B\equiv\log_{10}B$, and the distribution obeys $\int d(\log B)P(\log B)=1$. Using only standard wave field data, this prediction is a robust and rigorous way to test whether SGT is relevant [9,11,12]. Stochastic growth physics can coexist with a nonlinear process, which removes energy from the waves at high fields above a threshold B_c, with a revised prediction [9,10]

$$P(\log B)=(\sigma A\sqrt{2\pi})^{-1}(e^{-(\log B-\mu)^2/2\sigma^2} - e^{-(2\log B_c-\log B-\mu)^2/2\sigma^2}). \tag{3}$$

Here the normalization factor $A=\mathrm{erf}([\log B_c-\mu]/\sqrt{2}\sigma)$ involves the conventional error function.

In contrast, the uniform secular growth model predicts a uniform (flat) distribution $P(\log B)$ below B_c [9], since $\log B$ increases linearly with time for constant Γ, with differing distibutions above B_c for different nonlinear histories; e.g., exponential nonlinear growth/damping leads to a uniform distribution whereas trapping leads to a peak at a nonlinear level. SGT is also important as a member of a wide class of descriptions for instabilities in inhomogeneous systems characterized by different wave statistics. These differ due to varying degrees of randomness, inhomogeneity, and interactions between the ambient plasma, unstable waves, and driving distribution. In particular, whereas SGT involves lognormal statistics and a self-consistent wave/driving distribution system interacting in an independent, inhomogeneous plasma, SOC systems [15] have power-law distributions of properties (e.g., B) and involve the medium, waves, and unstable particles all undergoing mutually self-consistent interactions. Analysis of wave statistics thus allows the physics of the wave growth and interactions with the medium to be constrained.

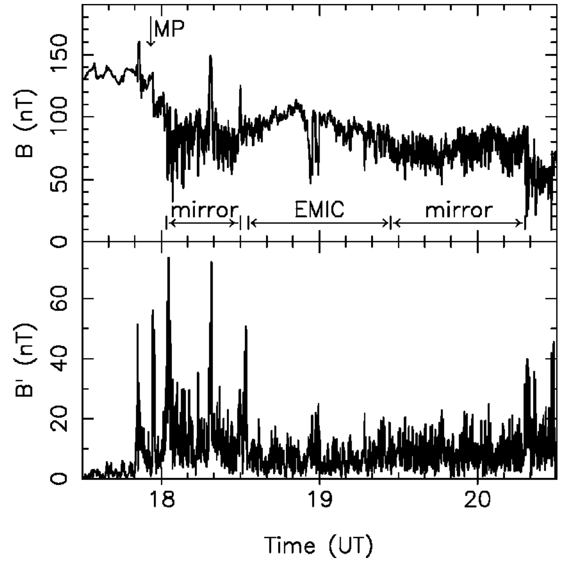

FIG. 2. Magnetic field amplitudes as a function of time for 6 October 1984 (day 280): (a) 6-s averaged data from the AMPTE magnetometer, (b) wave amplitudes B' described in the text. Labels show magnetopause crossings and periods with EMIC and mirror waves identified in Ref. [5].

The consistency of SGT with the statistics of mirror and EMIC waves in Earth's magnetosheath is next established by comparing the SGT predictions (2) and (3) with data from NASA's AMPTE-CCE spacecraft for the periods 1730–030 UT on 6 October 1984 (day 280) and 0210–0420 UT on 13 December 1984 (day 348). Anderson and Fuselier [5] discussed both periods in detail using magnetometer, proton, and electron data. They identified when EMIC and mirror waves were present, justified the mode identifications, and showed that the waves were correlated with variations in the proton temperature anisotropy. The SGT analysis presented here uses magnetic field vectors from the AMPTE magnetometer [16], averaged over the 6-s spacecraft period, that were obtained directly from the National Space Science Data Center. Figure 2(a) presents the magnetic field amplitude as a function of time, with labels showing where Anderson and

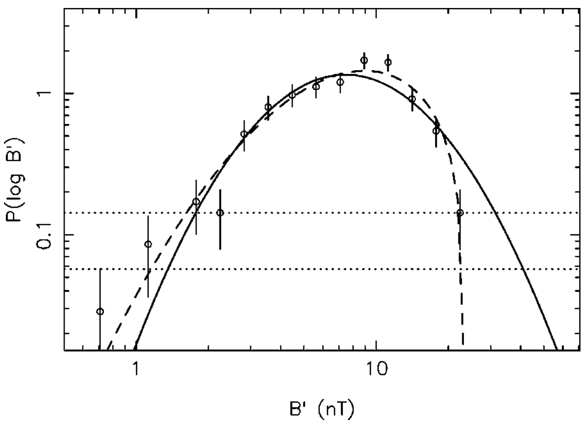

FIG. 3. Circle symbols with $\pm\sqrt{N}$ uncertainties show the observed probability distribution $P(\log B')$ for mirror waves during the period 1927–2018 UT on day 280, 1984. Dotted lines show the 2 and 5 count lines. Solid and dashed lines show best fits to the SGT predictions (2) and (3), respectively.

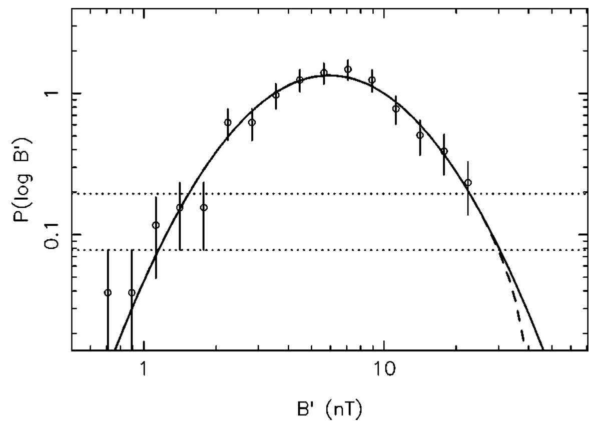

FIG. 4. The observed distribution $P(\log B')$ and best fits to the SGT predictions for EMIC waves during the period 0217–0250 UT on day 348, 1984, in Fig. 3's format.

Fuselier [5] identified magnetopause crossings and periods with EMIC and mirror waves. The figure shows bursty waves superposed on the varying background field $\mathbf{B}_b$. To avoid biasing the $P(\log B)$ distribution, trends in $\mathbf{B}_b$ are removed by subtracting a centered, 11 sample, sliding estimate of $\mathbf{B}_b$, and the resulting field strengths $B'=|\mathbf{B}-\mathbf{B}_b|$ are compared with SGT. Figure 2(b) shows that this procedure removes trends in $\mathbf{B}_b$ very well, and also provides clear evidence that EMIC and mirror waves are both intrinsically bursty with widely varying amplitudes. Similar results are found for day 348 (not shown).

Circle symbols in Fig. 3 show the observed distribution $P(\log B')$ for mirror waves during the period 1927–2018 UT (excising the subinterval 1946–1955, due to large-scale rotational discontinuities preventing accurate estimates of $\mathbf{B}_b$) on day 280 with 350 field samples. Error bars show $\pm\sqrt{N}$ uncertainties, while dotted lines are the two and five count levels. The solid curve shows the best fit to the prediction (2) for pure SGT, obtained by minimizing χ^2 using a geometric simplex method [17] for bins with more than two counts. Very good quantitative agreement is apparent. Table I summarizes the parameters and statistics of the fit, which has reasonable statistical significance: $\chi^2=26.2$ for ten degrees of freedom: the significance probability $P(\chi^2)$ (of obtaining a larger χ^2 even if the model is correct) is 0.35%. The dashed line shows the best fit to the prediction (3) for SGT with a coexisting nonlinear process at high fields. Now excellent quantitative and statistical agreement exists, with a higher significance probability $\sim$26%. Thus, Fig. 3 shows very strong evidence that SGT quantitatively accounts for the functional form of the EMIC wave statistics in this period, and that a nonlinear process is active at high wave fields $\geqslant 15$ nT.

TABLE I. Fits of SGT predictions to observational data.

Mode	Fit	μ	σ	$\log_{10}B_c$	χ^2	N	$P(\chi^2)$
Mirror	(2)	0.87 ± 0.10	0.29 ± 0.10		26	10	0.3%
Mirror	(3)	1.4 ± 0.5	0.42 ± 0.10	1.4 ± 0.1	11	9	26%
EMIC	(2)	0.77 ± 0.10	0.30 ± 0.05		5.7	11	89%
EMIC	(3)	0.77 ± 0.10	0.30 ± 0.05	1.6 ± 0.3	5.6	10	85%

Figure 4 shows the distribution $P(\log B)$ for the period 0217–0250 UT (excising the subinterval 0231–0233, as justified above for the mirror interval) on day 348, during which EMIC waves are present [5], for a total of 257 field samples. The format is identical to Fig. 3. Once again, fitting the SGT predictions (2) and (3) to the observed distribution leads to excellent quantitative and statistical agreement—$P(\chi^2)=89\%$ and 85%, respectively. Figure 4 thus shows very strong evidence that SGT accounts for the functional form of the wave statistics in this period, this time without significant evidence for an active nonlinear process at high fields

Analysis of other intervals during these days, during which EMIC and mirror mode waves are present [5,6], yield analagous results (not shown). Typically, higher statistical significances are found for shorter intervals free from obvious rotational discontinuities or other changes in plasma environment. The evidence for an active nonlinear process also varies from period to period, ranging from nonexistent to strong for either mode.

The observed distributions $P(\log B)$ in Figs. 3 and 4 for mirror and EMIC waves are thus strongly inconsistent with both the uniform secular model and with SOC, which predict uniform and power-law distributions, respectively. Instead, the observed distributions agree very well with the SGT predictions, with some evidence for nonlinear processes becoming active for EMIC and mirror waves and removing wave energy at fields above ≈ 15 nT. SGT thus qualitatively explains the burstiness, widely varying fields, and persistence of the waves (and their energy source) throughout the magnetosheath. In addition, these results represent a successful test of the expectation that SGT applies where a system is near marginal stability, since the mirror and EMIC waves both occur where independent measurements and theory show [5,7] that the observed proton distributions are close to marginal stability. Put another way, both the wave statistics and the observed particle distributions are consistent with the waves and particles being near marginal stability, confirming that an SGT state has developed near marginal stability and that the theoretical hypotheses underlying SGT are self-consistent.

Why might an SGT state be attained here? Observationally the linear instabilities producing the EMIC and mirror waves are driven by proton temperature anisotropies [5–7], with wave growth acting to decrease the temperature anisotropy. An important insight, however, is that the temperature anisotropy rebuilds as draped magnetic field lines are carried closer to the magnetopause and particles with low pitch angles and high parallel speeds (which contribute most to $T_{\parallel}$) move along the field and are lost, which decreases $T_{p\parallel}$ and increases the ratio $T_{p\perp}/T_{p\parallel}$. The wave-particle system thus naturally involves a competition between destruction (by wave growth) and rebuilding (by convection) of the free energy source in the inhomogeneous magnetosheath, as envisaged in other SGT models [9,11]. This qualitative model appears viable but needs to be extended to predict μ and σ.

In conclusion, this paper presents very strong evidence that SGT accounts for the form of the field statistics of both EMIC and mirror waves in Earth's magnetosheath and so provides a qualiitative theoretical explanation for their burstiness, widely varying fields, and persistence. The observed wave statistics are strongly inconsistent with both uniform secular growth and SOC. There is weak evidence that a nonlinear process coexists with SGT only at high fields $\geqslant 15$ nT. These results represent a successful test of SGT in a situation where the waves are known independently to be near marginal stability. Simultaneously, this is the first application of SGT to primarily magnetic waves (rather than high frequency, primarily electrostatic waves near f_p [9,11–13]), to waves driven by ions rather than electrons, and to a temperature anisotropy instability rather than a beam instability. SGT also applies in all six contexts considered thus far, ranging from the solar wind (type III solar bursts and thermal waves [9,13]), the edge and major volume of Earth's foreshock [11–13], to EMIC and mirror waves in Earth's magnetosheath. This implies that SGT should be considered widely applicable to space plasmas, where most waves have characteristics qualitatively indicative of SGT (burstiness, widely varying fields, and persistence), and so presumably to astrophysical plasmas.

The Australian Research Council and NASA Grant No. NAG5-6369 supported this research. We gratefully acknowledge constructive comments from P. A. Robinson and discussions with B. J. Anderson.

[1] W. Baumjohann and R.A. Treumann, *Basic Space Plasma Physics* (Imperial College Press, London, 1997).
[2] E.J. Smith, R.E. Holzer, M.G. McLeod, and C.T. Russell, J. Geophys. Res. **72**, 4803 (1967).
[3] P. Rodriguez, J. Geophys. Res. **84**, 917 (1979).
[4] B.T. Tsurutani *et al.*, J. Geophys. Res. **87**, 6060 (1982).
[5] B.J. Anderson and S.A. Fuselier, J. Geophys. Res. **98**, 1461 (1993).
[6] B.J. Anderson, S.A. Fuselier, S.P. Gary, and R.E. Denton, J. Geophys. Res. **99**, 5877 (1994).
[7] S.P. Gary *et al.*, J. Geophys. Res. **99**, 5903 (1994).
[8] P.A. Robinson, Sol. Phys. **139**, 147 (1992).
[9] P.A. Robinson, I.H. Cairns, and D.A. Gurnett, Astrophys. J. **407**, 790 (1993).
[10] P.A. Robinson, Phys. Plasmas **2**, 1466 (1995).
[11] I.H. Cairns and P.A. Robinson, Geophys. Res. Lett. **24**, 369 (1997).
[12] I.H. Cairns and P.A. Robinson, Phys. Rev. Lett. **82**, 3066 (1999).
[13] I.H. Cairns, P.A. Robinson, and R.R. Anderson, Geophys. Res. Lett. **27**, 61 (2000).
[14] T.H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
[15] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
[16] T.A. Potemra, L.J. Zanetti, and M.H. Acuna, IEEE Trans. Geosci. Remote Sens. **GE-23**, 246 (1985).
[17] W.H. Press *et al.*, *Numerical Recipes* (Cambridge University Press, New York, 1986).